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Signatures of finite representations of real, outer Lie algebras

Alexander N Rudy

Department of Mathematics, Belarussian State Polytechnical Academy, Scarina av. 65, Minsk, Republic of Belarus

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Abstract. The paper deals with the real Lie algebras of outer type and their arbitrary irreducible representations. Hermitian forms being invariant relative to these representations are considered. Signature formulae for these forms are obtained.

1. Introduction

Let $\mathfrak{g}$ be the simple complex Lie algebra of type A_r, D_r, E_6, i.e. the algebra with the non-trivial symmetry ε of its Dynkin diagram. Let $\mathfrak{g}_\sigma$ be any real form of outer type for $\mathfrak{g}$, i.e. $\mathfrak{g}_\sigma = \mathfrak{sl}_{r+1}(\mathbb{R}), \mathfrak{sl}_{(r+1)/2}(\mathbb{H})$, r is odd, $\mathfrak{so}_{2i+1,2r-2i-1}$, $i = 0, 1, \ldots, [(r-1)/2]$, EI, EIV. Consider an irreducible representation $\varphi : \mathfrak{g} \to \mathfrak{sl}(V)$. Let λ be the highest weight of the representation φ and let χ_λ be the character of the representation φ. Denote by $\dim V$ or $\dim(\lambda)$ the dimension of the representation φ. From [1] it follows that if $\varepsilon(\lambda) = \lambda$, then $\varphi(\mathfrak{g}) \subset \mathfrak{su}(p, q)$, where $p + q = \dim V$. Let $\delta_{\mathfrak{g}_\sigma}(\lambda) = p - q$. The formulae for $|\delta|$ in the case of A_r, D_r were obtained in [1]. As follows from this paper, it is possible to improve them and to obtain similar formulae in the case of EI and EIV.

The result can be used for the construction of mathematical models in theoretical physics.

2. Definitions

The definitions used in this paper coincide with those in [2, 3]. Let $\mathfrak{g}_\tau$ be the fixed compact real form of the algebra $\mathfrak{g}$ and let τ be the conjugation of the algebra $\mathfrak{g}$ with respect to $\mathfrak{g}_\tau$. Consider an involution θ of the algebra $\mathfrak{g}$ such that $\theta(\mathfrak{g}_\tau) = \mathfrak{g}_\tau$. Let $\sigma = \tau \circ \theta = \theta \circ \tau$. Denote by $\mathfrak{g}_\sigma$ the real form of the algebra $\mathfrak{g}$ such that σ is a conjugation of the algebra $\mathfrak{g}$ with respect to $\mathfrak{g}_\sigma$. The real form $\mathfrak{g}_\sigma$ is called the real form of outer type, if $\theta \in \mathrm{Aut}(\mathfrak{g}_\sigma) \setminus \mathrm{Int}(\mathfrak{g}_\sigma)$. Suppose $\mathfrak{t}$ is a Cartan subalgebra of $\mathfrak{g}_\tau$ such that $\theta(\mathfrak{t}) = \mathfrak{t}$, $\mathfrak{h}$ is a Cartan subalgebra of $\mathfrak{g}$ such that $\mathfrak{t}^{\mathbb{C}} = \mathfrak{h}$, and R is a root system associated with the pair $(\mathfrak{g}, \mathfrak{h})$. Let $B(\,,\,)$ be a Killing form of $\mathfrak{g}$, and let $(\,,\,) = -(1/(2\pi)^2)B(\,,\,)$ be a positive definite scalar product on $\mathfrak{t}$. Let $\alpha \in R$, by H_α denote an element of $\mathfrak{h}$ such that $B(H_\alpha, H) = \alpha(H)$ for all $H \in \mathfrak{h}$. Define the embedding $\psi : R \to \mathfrak{t}$ by $\psi(\alpha) = 2\pi\sqrt{-1}H_\alpha$ for all $\alpha \in R$. Suppose $\Pi = \{\alpha_1, \ldots, \alpha_r\}$ is a set of the simple roots of R, and $\{H_i\}_{i=1}^r$ is a basis of $\mathfrak{t}$ such that $(H_i, \alpha_j) = \delta_{ij}, i, j = 1, \ldots, r$. Let $(E_\alpha)_{\alpha \in R}$ be the Chevalley system for $(\mathfrak{g}, \mathfrak{h})$ [4]. Denote by θ_0 an automorphism generated by the non-trivial symmetry ε of its Dynkin diagram, i.e. $\theta_0(E_\alpha) = E_{\varepsilon(\alpha)}$ for all $\alpha \in \Pi$. If $\theta \notin \mathrm{Int}(\mathfrak{g}_\tau)$, then without loss of generality $\theta = \theta_0$ or

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$\theta = \theta_0 \circ \exp(\mathrm{ad}(H_{i_0}/2))$ for some $i_0, 1 \leqslant i_0 \leqslant r$. Let $R^\vee$ be the root system dual to R, i.e.

$$R^\vee = \left\{ \frac{2\alpha}{(\alpha,\alpha)} \middle| \alpha \in R \right\}$$

and let $\omega_i, i = 1, \dots, r$ be basis representations of the algebra $\mathfrak{g}$, i.e. $2(\omega_i, \alpha_k)/(\alpha_k, \alpha_k) = \delta_{ik}$, where $\alpha_k \in \Pi, i, k = 1, \dots, \mathrm{rank}(\mathfrak{g})$. Then $\lambda = \sum_{j=1}^{r} \lambda_j \omega_j$, where $\lambda_j, j = 1, \dots, r$ are the marks of λ. Denote by A_r^+, B_r^+, C_r^+ the set of all positive roots R^+ for the algebra $\mathfrak{g}$ of the type A_r, B_r, C_r, respectively.

The simple root enumerations coincide with those in [2, 3].

3. The case $\mathfrak{g} = \mathfrak{sl}(r+1, \mathbb{C}), r = 2n-1$

Let $\mathfrak{g}_\sigma = \mathfrak{sl}_n(\mathbb{H})$. Then $\theta = \theta_0$. Consider the representation

$$\begin{array}{cccccccccc} \lambda_1 & \lambda_2 & & \lambda_{n-1} & \lambda_n & \lambda_{n-1} & & \lambda_2 & \lambda_1 \\ 0 - & 0 - & \cdots & - 0 - & 0 - & 0 - & \cdots & - 0 - & 0 \end{array} \tag{1}$$

of the algebra $\mathfrak{sl}(2n, \mathbb{C})$. From [1] it follows that

$$|\delta| = \left(\frac{\dim V}{1 \times 3 \cdots (2n-1)} (\lambda_n + 1)(2\lambda_{n-1} + \lambda_n + 3) \right.$$
$$\left. \times \cdots (2\lambda_1 + \cdots + 2\lambda_{n-1} + \lambda_n + 2n - 1) \right)^{1/2} \tag{2}$$

where $\dim V$ is the dimension of the representation (1). Note that

$$\dim V = \frac{\prod_{\beta \in A_{2n-1}^+} (\beta, \lambda + \rho)}{\prod_{\beta \in A_{2n-1}^+} (\beta, \rho)}. \tag{3}$$

Consider the representation

$$\begin{array}{cccccc} \lambda_1 & \lambda_2 & & \lambda_{n-1} & & \lambda_n \\ 0 - & 0 - & \cdots & - 0 & \Rightarrow & 0 \end{array} \tag{4}$$

of the algebra $\mathfrak{so}(2n+1, \mathbb{C})$. Then, using straightforward calculations, we derive

$$\left(\prod_{\beta \in A_{2n-1}^+} (\beta, \lambda + \rho) \right) (\lambda_n + 1)(2\lambda_{n-1} + \lambda_n + 3) \cdots (2\lambda_1 + \cdots + 2\lambda_{n-1} + \lambda_n + 2n - 1)$$
$$= \left(\prod_{\beta \in B_n^+} (\beta, \lambda + \rho) \right)^2 2^{2n}. \tag{5}$$

Inserting equations (3) and (5) in (2) yields the formula

$$\left| \delta_{\mathfrak{sl}_n(\mathbb{H})} \right| = \dim \left(\begin{array}{cccccc} \lambda_1 & \lambda_2 & & \lambda_{n-1} & & \lambda_n \\ 0 - & 0 - & \cdots & - 0 & \Rightarrow & 0 \end{array} \right). \tag{6}$$

Let $\mathfrak{g}_\sigma = \mathfrak{sl}_{2n}(\mathbb{R})$. Then $\theta = \theta_0 \circ \exp(\mathrm{ad}(H_n/2))$. Consider the representation (1) of the algebra $\mathfrak{sl}(2n, \mathbb{C})$. From [1] it follows that if λ_n is odd, then $\delta = 0$. If λ_n is even, then

$$|\delta| = \left(\frac{\dim V \big(1 \times 3 \cdots (2n-1) \big)}{(\lambda_n + 1)(2\lambda_{n-1} + \lambda_n + 3) \cdots (2\lambda_1 + \cdots + 2\lambda_{n-1} + \lambda_n + 2n - 1)} \right)^{1/2} \tag{7}$$

where $\dim V$ is the dimension of the representation (1). Consider the representation (4) of the algebra $\mathfrak{so}(2n+1,\mathbb{C})$. Then, similarly, we derive

$$\frac{\prod_{\beta\in A^+_{2n-1}}(\beta,\lambda+\rho)}{(\lambda_n+1)(2\lambda_{n-1}+\lambda_n+3)\cdots(2\lambda_1+\cdots+2\lambda_{n-1}+\lambda_n+2n-1)} = \left(\prod_{\beta\in B^+_n,\,\beta=\beta^\vee}(\beta,\lambda+\rho)\right)^2. \tag{8}$$

Inserting equations (3) and (8) in (7) for even λ_n yields the formula

$$\left|\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}\right| = \frac{\prod_{\beta\in B^+_n,\,\beta=\beta^\vee}(\beta,\lambda+\rho)}{2!4!\cdots(2n-2)!}. \tag{9}$$

So the formula for δ in the case $\mathfrak{g}_\sigma=\mathfrak{sl}_{2n}(\mathbb{R})$ coincide completely with [2, equation (5)] in the case $\mathfrak{g}_\sigma=\mathfrak{so}_{1,2n}$. Hence from [2, theorem 1] and equations (6) and (9) we derive the following theorem.

Theorem 1. Let $\mathfrak{g}=\mathfrak{sl}(2n,\mathbb{C})$. Suppose $\lambda=\sum_{j=1}^{n-1}\lambda_j(\omega_j+\omega_{2n-j})+\lambda_n\omega_n$ is the highest weight of the representation $\varphi:\mathfrak{sl}(2n,\mathbb{C})\to\mathfrak{sl}(V)$. Let $\mathfrak{g}_\sigma=\mathfrak{sl}_n(\mathbb{H})$, then $\theta=\theta_0$ and

$$\left|\delta_{\mathfrak{sl}_n(\mathbb{H})}\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n&\lambda_{n-1}&&\lambda_2&\lambda_1\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\text{—}&0&\text{—}0\text{—}&\cdots&\text{—}0\text{—}&0\end{pmatrix}\right| = \dim\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\Rightarrow&0\end{pmatrix}.$$

Let $\mathfrak{g}_\sigma=\mathfrak{sl}_{2n}(\mathbb{R})$. Then $\theta=\theta_0\circ\exp(\mathrm{ad}(H_n/2))$ and

$$\begin{aligned}\left|\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n&\lambda_{n-1}&&\lambda_2&\lambda_1\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\text{—}&0&\text{—}0\text{—}&\cdots&\text{—}0\text{—}&0\end{pmatrix}\right| &= \left|\chi_{\begin{smallmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\Rightarrow&0\end{smallmatrix}}(H_n/2)\right|\\ &= \left|\delta_{\mathfrak{so}_{1,2n}}\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\Rightarrow&0\end{pmatrix}\right|.\end{aligned}$$

Moreover, if λ_n is odd, then $\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}=0$. If λ_n is even, then

$$\left|\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}\right| = \frac{\prod_{\beta\in B^+_n,\,\beta=\beta^\vee}(\beta,\lambda+\rho)}{2!4!\cdot\cdots(2n-2)!}.$$

Corollary. Let λ_n be even. Then

$$\left|\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}\right| = \left|\delta_{\mathfrak{so}_{1,2n}}\right| = \frac{\dim\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n&\lambda_{n-1}&&\lambda_2&\lambda_1\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\text{—}&0&\text{—}0\text{—}&\cdots&\text{—}0\text{—}&0\end{pmatrix}}{\dim\begin{pmatrix}\lambda_1&\lambda_2&&\lambda_{n-1}&\lambda_n\\ 0\text{—}&0\text{—}&\cdots&\text{—}0\Rightarrow&0\end{pmatrix}}.$$

Proof. From equations (2) and (7) we find

$$\left|\delta_{\mathfrak{sl}_{2n}(\mathbb{R})}\right|\cdot\left|\delta_{\mathfrak{sl}_n(\mathbb{H})}\right| = \dim V.$$

The result follows immediately from theorem 1.

4. The case $\mathfrak{g} = \mathfrak{sl}(r+1, \mathbb{C})$, $r = 2n$

Let $\mathfrak{g}_\sigma = \mathfrak{sl}_{2n+1}(\mathbb{R})$. Then $\theta = \theta_0$. Discussing this as in the previous case we derive, using the results from [1], the following theorem.

Theorem 2. Let $\mathfrak{g}_\sigma = \mathfrak{sl}_{2n+1}(\mathbb{R})$. Consider the representation

$$\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_n}{0}-\overset{\lambda_n}{0}-\cdots-\overset{\lambda_2}{0}-\overset{\lambda_1}{0}$$

of the algebra $\mathfrak{g} = \mathfrak{sl}(2n+1, \mathbb{C})$ and the representation

$$\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{n-1}}{0}\Leftarrow\overset{\lambda_n}{0}$$

of the algebra $\mathfrak{sp}(2n, \mathbb{C})$. Then

$$\left|\delta_{\mathfrak{sl}_{2n+1}(\mathbb{R})}\left(\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_n}{0}-\overset{\lambda_n}{0}-\cdots-\overset{\lambda_2}{0}-\overset{\lambda_1}{0}\right)\right| = \dim\left(\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{n-1}}{0}\Leftarrow\overset{\lambda_n}{0}\right).$$

5. The case $\mathfrak{g} = \mathfrak{so}(2r, \mathbb{C})$, $r \geqslant 4$

The Dynkin diagram for $\mathfrak{so}(2r, \mathbb{C})$ is

$$\begin{array}{l} \overset{\alpha_1}{0}-\overset{\alpha_2}{0}-\cdots-\overset{\alpha_{r-2}}{0}-\overset{\alpha_{r-1}}{0} \\ \qquad\qquad\qquad\quad\;\; | \\ \qquad\qquad\qquad\quad\;\; 0\alpha_r. \end{array}$$

Consider the symmetry ε such that

$$\varepsilon(\alpha_1) = \alpha_1, \quad \varepsilon(\alpha_2) = \alpha_2, \quad \ldots, \quad \varepsilon(\alpha_{r-2}) = \alpha_{r-2}, \quad \varepsilon(\alpha_{r-1}) = \alpha_r, \quad \varepsilon(\alpha_r) = \alpha_{r-1}.$$

Suppose $\mathfrak{g}_\sigma = \mathfrak{so}_{1,2r-1}$. Then $\theta = \theta_0$. Consider the representation

$$\begin{array}{l} \overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{r-2}}{0}-\overset{\lambda_{r-1}}{0} \\ \qquad\qquad\qquad\quad\;\; | \\ \qquad\qquad\qquad\quad\;\; 0\lambda_{r-1} \end{array} \tag{10}$$

of the algebra $\mathfrak{so}(2r, \mathbb{C})$ and the representation

$$\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{r-2}}{0}\Leftarrow\overset{\lambda_{r-1}}{0} \tag{11}$$

of the algebra $\mathfrak{sp}(2r-2, \mathbb{C})$. Then, using the results from [1], we derive

$$\left|\delta_{\mathfrak{so}_{1,2r-1}}\left(\begin{array}{l} \overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{r-2}}{0}-\overset{\lambda_{r-1}}{0} \\ \qquad\qquad\qquad\quad\;\; | \\ \qquad\qquad\qquad\quad\;\; 0\lambda_{r-1} \end{array}\right)\right| = \dim\left(\overset{\lambda_1}{0}-\overset{\lambda_2}{0}-\cdots-\overset{\lambda_{r-2}}{0}\Leftarrow\overset{\lambda_{r-1}}{0}\right).$$

Let $\mathfrak{g}_\sigma = \mathfrak{so}_{2i+1,2r-2i-1}$, $i = 1, \ldots, [(r-1)/2]$. Then $\theta = \theta_0 \circ \exp(\mathrm{ad}(H_i/2))$. From [1] it follows that

$$|\delta| = \frac{|\det(\zeta_{pq})|}{2^{2i(r-i-1)}(0!2!\cdots(2r-2i-4)!)(1!3!\cdots(2i-1)!)(1\times 3\cdots(2r-2i-3))} \tag{12}$$

where $\det(\zeta_{pq})$ denotes an $(r-1)\times(r-1)$ determinant whose pq element is ζ_{pq} and

$$\zeta_{pq} = (-1)^{h_q} h_q^{2p-1} \qquad p = 1, \ldots, i \quad q = 1, \ldots, r-1$$

$$\zeta_{pq} = h_q^{2(p-i)-1} \qquad p = i+1, \ldots, r-1 \quad q = 1, \ldots, r-1$$

$$h_q = \lambda_q + \lambda_{q+1} + \cdots + \lambda_{r-1} + r - q \qquad q = 1, \ldots, r-1.$$

Note that equation (12) in the case $\mathfrak{g}_\sigma = \mathfrak{so}_{2i+1,2r-2i-1}$ coincide completely with [2, equation (20)] in the case $\mathfrak{g}_\sigma = \mathfrak{sp}_{i,r-i-1}$. The foregoing proves the theorem.

Theorem 3. Let $\mathfrak{g} = \mathfrak{so}(2r, \mathbb{C})$. Suppose $\lambda = \sum_{j=1}^{r-2} \lambda_j \omega_j + \lambda_{r-1}(\omega_{r-1} + \omega_r)$ is the highest weight of the representation $\varphi : \mathfrak{so}(2r, \mathbb{C}) \to \mathfrak{sl}(V)$. Let $\mathfrak{g}_\sigma = \mathfrak{so}_{1,2r-1}$, then $\theta = \theta_0$ and

$$\left|\delta_{\mathfrak{so}_{1,2r-1}}\left(\begin{array}{cccc} \lambda_1\ \ \lambda_2 & & \lambda_{r-2}\ \lambda_{r-1} \\ 0\!-\!0\!- & \cdots & -0\!-\!0 \\ & & | \\ & & 0\lambda_{r-1} \end{array}\right)\right| = \dim\left(\begin{array}{ccc} \lambda_1\ \ \lambda_2 & & \lambda_{r-2}\ \lambda_{r-1} \\ 0\!-\!0\!- & \cdots & -0 \Leftarrow 0 \end{array}\right).$$

Let $\mathfrak{g}_\sigma = \mathfrak{so}_{2i+1,2r-2i-1}$, $i = 1, \ldots, [(r-1)/2]$, then $\theta = \theta_0 \circ \exp(\mathrm{ad}(H_i/2))$ and

$$\left|\delta_{\mathfrak{so}_{2i+1,2r-2i-1}}\left(\begin{array}{cccc} \lambda_1\ \ \lambda_2 & & \lambda_{r-2}\ \lambda_{r-1} \\ 0\!-\!0\!- & \cdots & -0\!-\!0 \\ & & | \\ & & 0\lambda_{r-1} \end{array}\right)\right| = \left|\chi_{\begin{array}{ccc} \lambda_1\ \lambda_2 & & \lambda_{r-2}\ \lambda_{r-1} \\ 0\!-\!0\!- & \cdots & -0 \Leftarrow 0 \end{array}}(H_i/2)\right|$$

$$= \left|\delta_{\mathfrak{sp}_{i,r-i-1}}\left(\begin{array}{ccc} \lambda_1\ \ \lambda_2 & & \lambda_{r-2}\ \lambda_{r-1} \\ 0\!-\!0\!- & \cdots & -0 \Leftarrow 0 \end{array}\right)\right|.$$

From theorem 3 it follows that it is possible to use the formulae for $\delta_{\mathfrak{sp}_{i,r-i-1}}$ from [2, 3] to calculate $\delta_{\mathfrak{so}_{2i+1,2r-2i-1}}$.

6. The case $\mathfrak{g} = E_6$

The Dynkin diagram for the algebra E_6 is

$$\begin{array}{ccccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_5 & \alpha_6 \\ 0 & - 0 - & 0 & - 0 - & 0 \\ & & | & & \\ & & 0\alpha_4. & & \end{array}$$

Consider the representation

$$\begin{array}{ccccc} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 \\ 0 & - 0 - & 0 & - 0 - & 0 \\ & & | & & \\ & & 0\lambda_4 & & \end{array}$$

of the algebra E_6 and the representation

$$\begin{array}{c} \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \\ 0 - 0 \rangle 0 - 0 \end{array}$$

of the algebra F_4. For $\mathfrak{g} = E_6$ the results are similar to the previous cases.

Theorem 4. Let $\mathfrak{g} = E_6$. Suppose $\lambda = \lambda_1(\omega_1 + \omega_6) + \lambda_2(\omega_2 + \omega_5) + \lambda_3\omega_3 + \lambda_4\omega_4$ is the highest weight of the representation $\varphi : E_6 \to \mathfrak{sl}(V)$. Let $\mathfrak{g}_\sigma = EIV$, then $\theta = \theta_0$ and

$$\left| \delta_{EIV} \left(\begin{array}{c} \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_2 \quad \lambda_1 \\ 0 - 0 - 0 - 0 - 0 \\ | \\ 0\lambda_4 \end{array} \right) \right| = \dim \left(\begin{array}{c} \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \\ 0 - 0 \rangle 0 - 0 \end{array} \right).$$

Let $\mathfrak{g}_\sigma = EI$, then $\theta = \theta_0 \circ \exp(\mathrm{ad}(H_4/2))$ and

$$\left| \delta_{EI} \left(\begin{array}{c} \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_2 \quad \lambda_1 \\ 0 - 0 - 0 - 0 - 0 \\ | \\ 0\lambda_4 \end{array} \right) \right| = \left| \delta_{FII} \left(\begin{array}{c} \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \\ 0 - 0 \rangle 0 - 0 \end{array} \right) \right|.$$

From theorem 4 it follows that it is possible to calculate δ_{EI} using table 2 of the signatures $|\delta_{FII}|$ from [3]. For example, if the marks λ_3 and λ_4 are odd, then $\delta_{EI} = 0$.

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